Problem Set 1: Structure of the Earth

INSTRUCTIONS: Answer the questions in bold. Turn in all your work (if you use a spreadsheet or matlab file, you may submit those by email). There are 4 questions, all equally weighted. You are free to work on the problems together, but written explanations (Question 4) must be individually done in your own words.

For a sphere of uniform density rotating around an axis that passes through its center, the moment of inertia (I) is:

$$I = \frac{2}{5}Mr^2 \tag{1}$$

where M is the mass of the sphere and r is the radius. Mantle peridotite exposed on earth's surface has a density of about 3300 kg/m^3. The radius of the earth is 6371 km. We are going to ignore the crust for the purposes of this problem set (it is volumetrically insignificant).

1. What is the moment of inertia for a spherical earth-sized planet composed completely of peridotite?

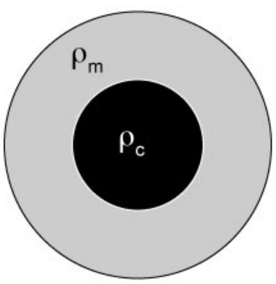

Figure 1: *2-layer earth*

Observations of earth's rotational velocity indicate that the true moment of inertia (I_e) is approximately $0.3308 M_e r_e^2$ (less than the value you calculated for # 1.) So, the assumption of uniform density does not hold for the planet earth, we need at least two layers of different densities (Fig. 1). Without knowing the density or size of the core, it is possible to estimate both using the moment of inertia (I_e) and mass of the earth ($M_e = 5.976 \times 10^{24} kg$) estimated from its rotation.

2. Solve for the a) size and b) density of the core

Moment of inertia is additive, meaning that you can treat the earth as the sum of two bodies. The same goes for mass. To save you a little time - the formula for the moment of inertia of a spherical shell is a messy polynomial and it will be difficult to algebraically separate the unknown terms. So, I suggest you use an approach shown in Fig. 2.

Instead of separating the core as a sphere and the mantle as a hollow shell, the same answer can be calculated if you treat the earth as the sum of two spheres. One is the earth-sized, uniform peridotite sphere from # 1 and the other is a sphere of the size of the core, whose density is the difference between core and mantle density ($\rho_\Delta = \rho_c - \rho_m$).

3. Given the true size of the core ($r_c = 3000km$), what are the average densities of the core and mantle?

4. Describe how your calculated values from #2 differ from the value given in the textbook. Can the difference in densities between the inner and outer core explain the disparity? How will the moment of inertia of the earth evolve as the outer core freezes onto the inner core over geologic time?

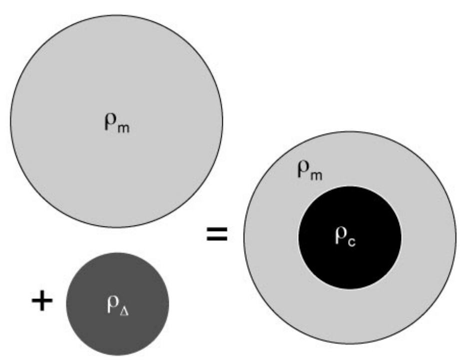

Figure 2: *Recommended approach*