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A RELATION BETWEEN THE DRIVING FORCE AND GEOID ANOMALY ASSOCIATED WITH MID-OCEAN RIDGES

BARRY PARSONS

Department of Earth and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139 (U.S.A.)

and

FRANK M. RICHTER

Department of the Geophysical Sciences, University of Chicago, Chicago, IL 60637 (U.S.A.)

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The driving force and geoid anomaly associated with the thermal structure of the oceanic plates are shown to be proportional to the first moment of the density structure with respect to depth and, hence, to each other. Both quantities exhibit the same functional dependence on age and this is given for two different thermal models. For the plate model the geoid anomaly and ridge driving force only increase slowly for ages greater than 40 m.y. in contrast to the half-space boundary layer model where a linear dependence on age holds for all ages. Isolation of the geoid anomaly related to the thermal structure of the plates would provide a direct measure of the magnitude of the ridge driving force.

1. Introduction

The difficulty of dealing explicitly with the mechanical properties of the lithosphere has restricted attempts to obtain the driving mechanism of plate tectonics as part of a coherent convective system. One approach that has been taken is to isolate the lithosphere or rigid mechanical boundary layer and consider a balance of the resulting forces applied along its boundaries and over its base by the rest of the system [1–3]. The body forces that are generally included are those to which attention is directed by association with major features such as ridges and subducting slabs. This approach will only be fruitful provided that buoyancy forces distributed within the mantle outside the plates do not make a significant contribution to maintaining plate motions. The resisting forces associated with a return mass flow, i.e. shear stresses and non-hydrostatic pressure forces, can be calculated using a consistent fluid mechanical model and included in the force balance [4]. This method is also useful in answering further questions such as the conditions necessary to create new subduction zones now and in the past [5].

One of the important driving terms in a force balance model is that associated with excess elevation and lower densities at mid-ocean ridges. The ridge driving force has been discussed by a number of authors [6–8]. In particular, Lister [8] calculated the force using a thermal boundary layer model of the plates. We present a brief derivation of the ridge force below and obtain a simple expression in terms of the first moment of differences in density structure. It has also been shown recently that the geoid anomalies associated with compensated topography can be calculated to good approximation as due to the dipole moment equivalent to the density distribution [9–11]. Hence the driving force and geoid anomaly associated with the thermal structure of mid-ocean ridges are shown to be simply proportional to one another. If the latter can be obtained from satellite altimeter observations of the geoid the ridge driving force can be obtained directly. The age dependence of either variable is considered for two thermal models. In the case of the plate model the driving force increases only slowly for sea floor older than 40 m.y., in distinction to a half-space model in which the force continues to increase linearly with age. The two thermal models thus give rather different magnitudes and distributions for the ridge driving force.

2. The driving force associated with mid-ocean ridges

This force results from the fact that the excess elevation at ridge crests produces a pressure distribution such that the pressure at any given depth above the level of compensation is greater than that under older ocean floor. At the compensation depth the effect of the excess elevation balances the effect of the lower densities due to higher temperatures under the ridge crest, providing a reference level on which pressures are constant. Calculations of the variation in mean depth as a function of age using models for the thermal structure explicitly assume that such a reference level exists. Justification for this is provided by the similarity of the depth versus age relation in different ocean basins [12], which suggests that the non-hydrostatic pressure, and also the shear stresses, acting at the base of the plate are small [13]. The driving force considered here is that due to the hydrostatic pressure field alone when non-hydrostatic stresses are neglected.

Fig. 1 illustrates the geometry used to derive the ridge pushing force. The density depends on position only between the top and bottom surfaces, $z = h(x)$ and $z = l(x)$. We consider the forces acting on the three bounding surfaces. The force produced by the horizontal component of the normal stresses acting on the bottom boundary is:

$$F_1 = \int_{z=l(x)} P(x, z)\, \hat{n} \cdot \hat{i}\, \mathrm{d}s \tag{1}$$

where $P(x, z)$ is the pressure, $\hat{n}$ is a unit vector normal to the boundary $z = l(x)$ pointing inward, $\mathrm{d}s$ is an element of length along the boundary, and $\hat{i}$ is a unit vector in the x-direction. From the geometry of

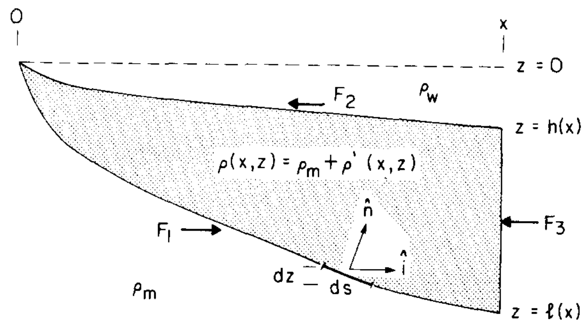

Fig. 1. Sketch of the geometry used to derive the ridge pushing force. The densities referred to are: ρ_m, the mantle density; ρ_w, the density of seawater; $\rho'(x, z)$, the laterally varying density produced by the temperature structure. Depths are measured downward from the ridge crest.

Fig. 1:

$$\hat{n} \cdot \hat{i} = \left.\frac{\mathrm{d}z}{\mathrm{d}s}\right|_{z=l(x)}$$

Then equation (1) becomes:

$$F_1 = \int_{z=l(x)} P(x, z)\, \mathrm{d}z$$

In the region where the density is everywhere equal to ρ_m, and hence on $z = l(x)$, the pressure is independent of x and depends only on the depth at which it is measured. The pressure on the boundary is:

$$P(x, z)|_{z=l(x)} = P(0, z) = \rho_w d_w g + \rho_m g z$$

where d_w is the depth at the ridge crest and g is the gravitational acceleration. The force is given by:

$$F_1 = \int_0^{l(x)} P(0, z)\, \mathrm{d}z = \rho_w g d_w l + \rho_m g \frac{l^2}{2} \tag{2}$$

The force, F_2, acting on the upper boundary can be derived in a similar fashion giving:

$$F_2 = \rho_w g d_w h + \rho_w g \frac{h^2}{2} \tag{3}$$

The final force, F_3, exerted by pressures on the vertical bounding surface is given by:

$$F_3 = \int_{h(x)}^{l(x)} P(x, z)\, \mathrm{d}z$$

where:

$$P(x, z) = \rho_w g d_w + \rho_m g l(x) - g \int_z^{l(x)} \rho(x, z')\,dz'$$

so that:

$$F_3 = \rho_w g d_w (l - h) + \rho_m g \frac{(l^2 - h^2)}{2} - g \int_h^l \int_z^l \rho'(x, z')\,dz'\,dz$$

The order of integration in the double integral can be exchanged to obtain a simpler expression for F_3:

$$F_3 = \rho_w g d_w (l - h) + \rho_m g \frac{(l^2 - h^2)}{2} - g \int_h^l (z' - h)\,\rho'(x, z')\,dz' \qquad (4)$$

The force required at x to balance the resulting force pushing out from the ridge is:

$$F_R = F_1 - F_2 - F_3$$

$$= (\rho_m - \rho_w) \frac{gh^2}{2} + g \int_h^l (z' - h)\,\rho'(x, z')\,dz' \qquad (5)$$

When x is in the interior of the plate this force must be provided by internal stresses in the plate, but when x is on a plate boundary an external force is necessary to balance the ridge pushing force. The vertical temperature structure and hence the depth relative to the ridge, $h(x)$, and the density structure, $\rho'(x, z)$, depend only on the age of the plate. Hence we can replace x by the equivalent age in evaluating (5) and consider F_R as a function of the age of the plate.

The above simple derivation is possible because we assume that the density distribution, or rather the temperature structure, is independently known. This is obtained by assuming a given velocity distribution, in this case rigid plate motion. The complete problem involves calculating the temperature structure and velocity field for chosen rheological properties and boundary conditions. The stress distribution, including non-hydrostatic terms, could then be obtained. By making reasonable assumptions about the velocity and density distributions, and neglecting non-hydrostatic stresses, a first approximation to the ridge driving force can be obtained without reference to material properties and boundary conditions on the rest of the system.

3. A geoid anomaly over mid-ocean ridges

Recent work [9–11] has shown that the geoid anomaly associated with locally compensated topography is given to a good approximation by the effect of a mass dipole equivalent to the compensating mass distribution. The geoid height at any distance from the ridge crest relative to that at the ridge crest would then be given by:

$$N = -\frac{2\pi G}{g} \int_0^l z\,\Delta\rho(x, z)\,dz$$

$$= -\frac{3}{2\bar{\rho}R} \int_0^l z\,\Delta\rho(x, z)\,dz \qquad (6)$$

where $\Delta\rho(x, z)$ is the difference in density structure at x from that at the ridge crest, $\bar{\rho}$ is the mean density of the Earth, G is the gravitational constant and R is the Earth's radius. In the notation of Fig. 1 the geoid anomaly is:

$$N = -\frac{3}{2\bar{\rho}R} \left[(\rho_m - \rho_w) \frac{h^2}{2} + \int_h^l (z - h)\,\rho'(x, z)\,dz \right] \qquad (7)$$

The above method of calculating the geoid anomaly for locally compensated topography is an approximation. Its range of validity can be examined by considering surface topography varying as $h \cos kx$. This is equivalent to a surface density distribution $\rho h \cos kx$ which we suppose to be compensated by a surface density distribution of opposite sign concentrated at a depth d. The exact geoid anomaly associated with this locally compensated topography is:

$$N = \frac{2\pi G}{g} \rho h \cos(kx) \left[\frac{1 - \exp(-kd)}{k} \right] \qquad (8)$$

whereas the dipole approximation equivalent to using

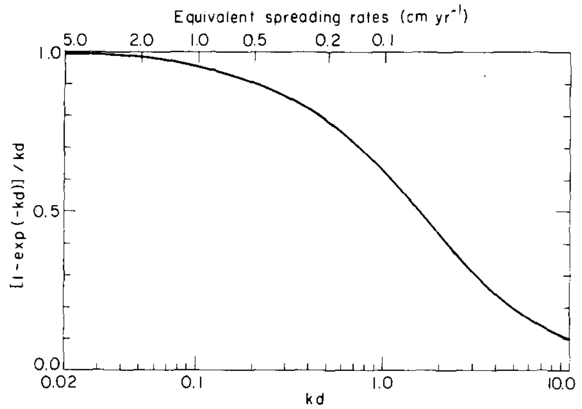

Fig. 2. The ratio of expressions given by (8) and (9) as a function of non-dimensional wave number, kd. The top scale gives the velocity equivalent to any value of kd using equation (14) and the same physical parameters listed in the caption to Fig. 3.

(6) is:

$$N = \frac{2\pi G}{g}\rho h d\cos(kx) \tag{9}$$

The ratio of the expressions given by (8) and (9) is plotted in Fig. 2. It can be seen that the dipole approximation is a good one when $kd \ll 1$. Hence in order for (7) to be valid the wavelengths associated with the density structure $\rho'(x, z)$ must be long compared to the plate thickness. Below it will be shown that this is the case for the thermal structure given by a plate model with moderate spreading velocities.

4. A relation between driving force and geoid anomaly

From (5) and (7) it can be seen that the geoid anomaly, N, and the pushing force, F_R, for the same-age sea floor are proportional to one another, the relation between them being:

$$F_R = -\left(\frac{2\bar{\rho}gR}{3}\right)N \tag{10}$$

In particular the dependence of F_R and N on age can be investigated together. We evaluate (5), and hence (7), for two different thermal models. In the plate model the thermal structure is calculated for a plate of constant thickness with a constant temperature maintained along the bottom boundary and on the side boundary beneath the ridge crest. The temperature structure is then:

$$T = T_m\left[\frac{z}{a} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi z}{a}\right)\exp\left(-\beta_n\frac{x}{a}\right)\right] \tag{11}$$

with

$$\beta_n = [(R^2 + n^2\pi^2)^{1/2} - R]\,, \quad R = ua/2\kappa$$

and T_m is the bottom boundary temperature, a is the plate thickness, u the spreading velocity and κ the thermal diffusivity (e.g. [12]). Here z is measured downward from the top of the plate. The equation of state for the density is taken to be:

$$\rho(x, z) = \rho_m\{1 + \alpha[T_m - T(x, z)]\} \tag{12}$$

where α is the volume expansion coefficient. Hence the geoid height over ocean floor of age t relative to that at the ridge is:

$$N = -\frac{3\alpha\rho_m}{2\bar{\rho}R}\int_0^a z\,[T_m - T(ut, z)]\,dz$$
$$= -\frac{\alpha\rho_m T_m a^2}{4\bar{\rho}R}\left[1 - \frac{12}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\exp\left(\frac{-\beta_n ut}{a}\right)\right] \tag{13}$$

only terms of first order in (αT_m) being retained. The primary contribution comes from the term for $n = 1$. This term has a Fourier transform:

$$\mathcal{F}[\exp(-\beta_1|x|/a)] = \frac{1}{\sqrt{2\pi}}\frac{2\beta_1 a}{\beta_1^2 + k^2a^2}$$

so that β_1/a represents a characteristic wave number associated with the density structure. It was shown above that (6) is a good approximation to the geoid anomaly for $kd \ll 1$. Here, with a choice of $d = a/2$ for the mean depth of compensation, the requirement is that:

$$kd = \frac{\beta_1}{a}\times\frac{a}{2} \simeq \frac{\pi^2\kappa}{2ua} \ll 1 \tag{14}$$

This will be satisfied if the spreading velocity is sufficiently large. The velocity scale at the top of Fig. 2 gives the spreading velocities equivalent to the values of kd on the bottom axis. The criterion is satisfied for velocities of 1 cm/yr or greater.

An alternative temperature structure is given by a half-space cooling model:

$$T = T_m \operatorname{erf} \frac{z}{2\sqrt{\kappa t}} \tag{15}$$

This solution is asymptotically equal to that for the plate model for small times. When applied to the calculation of depth versus age either of the solutions (11) or (15) give good agreement for ages less than 70 m.y. [12,14], but the temperature structure given by (11) also fits the variations in mean depth for older ocean floor [12]. Haxby and Turcotte [10] have given the geoid height relative to that at the ridge obtained from (15):

$$N = -\frac{3\alpha\rho_m}{2\bar{\rho}R} \int_0^\infty z\,[T_m - T(t, z)]\,dz$$

$$= -\frac{3\alpha\rho_m T_m \kappa t}{2\bar{\rho}R} \tag{16}$$

The ridge pushing force equivalent to (16) is:

$$F_R = g\alpha\rho_m T_m \kappa t \tag{17}$$

which is the result obtained by Lister [8]. The geoid heights relative to the ridge and the equivalent ridge force for both expressions (13) and (16) are plotted in Fig. 3 as a function of age. For ages less than 40 m.y. both solutions give the same linear dependence, the slopes being:

$$\frac{dN}{dt} = -0.15 \text{ m per m. y}$$

$$\frac{dF_R}{dt} = 3.6 \times 10^{10} \text{ N/m per m. y}. \tag{18}$$

This linear relation continues for the half-space model, but for the plate model the geoid anomaly and ridge pushing force change much more slowly for ages greater than 40 m.y. At an age of 200 m.y. the plate model gives a total variation of 14.0 m and 3.2×10^{12} N/m compared to 30.7 and 7.2×10^{12} N/m for the half-space model. The maximum geoid anomaly produced by the plate model from (13) is:

$$N_{max} = -\frac{\alpha\rho_m T_m a^2}{4\bar{\rho}R} \tag{19}$$

or 14.58 m. The equivalent ridge pushing force is 3.43×10^{12} N/m.

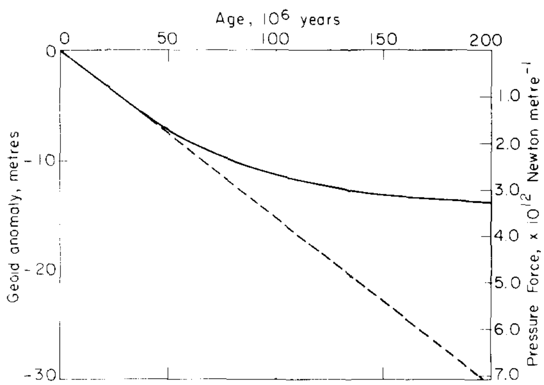

Fig. 3. The ridge driving force, and equivalent geoid anomaly associated with the thermal structure of the plate, given as a function of age. Physical parameters used to evaluate (13) and (17) are $\alpha = 3.3 \times 10^{-5}$ °C^{-1}, a = 120 km, $\kappa = 8.0 \times 10^{-7}$ m^2/s, T_m = 1300°C, ρ_m = 3.33 Mg/m^3, $\bar{\rho}$ = 5.53 Mg/m^3, and R = 6378 km.

5. Discussion

It has been shown that the driving force and geoid anomaly associated with the excess elevation and density structure of spreading ridges are simply proportional to one another. A simple physical explanation for this relation can be obtained by considering a mass column above a given depth as it moves away from the ridge. The plate cools, and the center of gravity of the column increases in depth. The ridge pushing force can be obtained by equating the release in gravitational potential energy due to the movement of the center of gravity to the work done against the external force balancing the ridge pushing force. The geoid height, in the approximation considered, is proportional to the dipole moment of the mass in the column. This is simply a product of the mass of the column and the depth of the center of gravity. Both N and F_R are proportional to changes in depth of the center of gravity and hence are proportional to one another.

The force required to balance the ridge pushing force is a function of age only. In the interior of the plate this is provided by internal stresses within the plate. At a subducting plate boundary an external force is necessary whose magnitude depends on the age of the plate when it subducts. For the plate model the ridge pushing force varies increasingly slowly for ages greater than 40 m.y. Thus the magnitude of the force entered into a force balance on the plate is only sensitive to the age of the subducting material when this is less than 40 m.y. The above considerations primarily apply to a plate with a simple spreading history i.e. with no major reorientations in spreading direction. For a plate with a complex spreading history the differential force, given by the derivative with respect to t of the expression for F_R equivalent to (13), acting perpendicular to isochrons, must be summed over the plate. The total force will still depend on the age of the subducting material but its direction will no longer be simply oriented perpendicular to the present ridge.

The geoid anomaly associated with the thermal structure of the plates has the same age dependence as the ridge driving force. It can be seen that the way the geoid varies with age is more sensitive to the presence of a bottom boundary condition than variations in depth with age. Attempts to extract this isostatic geoid anomaly from the total geoid obtained from satellite altimeter measurements are still at an early stage [15], but there are some indications that it has the behavior predicted by equation (13). If this part of the geoid can be successfully isolated it would provide a direct measurement of one of the driving forces contributing to the balance of forces on the plates.

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References

1 F.M. Richter, Dynamical models for sea floor spreading, Rev. Geophys. Space Phys. 11 (1973) 223–287.
2 D. Forsyth and S. Uyeda, On the relative importance of the driving forces of plate motion, Geophys. J.R. Astron. Soc. 43 (1975) 163–200.
3 W.M. Chapple and T.E. Tullis, Evaluation of the forces that drive the plates, J. Geophys. Res. 82 (1977) 1967–1984.
4 F.M. Richter and D. McKenzie, Simple plate models of mantle convection. J. Geophys. 44 (1978) 441–471.
5 B. Parsons, and D. McKenzie, Conditions for the creation of new trenches (in preparation).
6 W.R. Jacoby, Instability in the upper mantle and global plate movements, J. Geophys. Res. 75 (1970) 5671–5680.
7 E.V. Artyushkov, Stresses in the lithosphere caused by crustal thickness inhomogeneities, J. Geophys. Res. 78 (1973) 7675–7708.
8 C.R.B. Lister, Gravitational drive on oceanic plates caused by thermal contraction, Nature 257 (1975) 663–665.
9 J.R. Ockendon and D.L. Turcotte, On the gravitational potential and field anomalies due to thin mass layers, Geophys. J.R. Astron. Soc. 48 (1977) 479–492.
10 W.F. Haxby and D.L. Turcotte, On isotatic geoid anomalies, J. Geophys. Res. 83 (1978) 5473–5478.
11 D.L. Turcotte, and J.R. Ockendon, Geoid anomalies and the near-surface dipole distribution of mass, Proc. 9th GEOP Conf., Dept. Geodet. Sci. Rep. No. 280, Ohio State Univ., Columbus, Ohio (1978) 257–260.
12 B. Parsons and J.G. Sclater, An analysis of the variation of ocean floor bathymetry and heat flow with age, J. Geophys. Res. 82 (1977) 803–827.
13 B. Parsons and F.M. Richter, Mantle convection and the oceanic lithosphere, in: "The Sea", Vol. 7, C. Emiliani, ed. (John Wiley and Sons, New York, N.Y.) in press.
14 E.E. Davis and C.R.B. Lister, Fundamentals of ridge crest topography, Earth Planet. Sci. Lett. 21 (1974) 405–413.
15 W.F. Haxby, The mid-ocean ridge geoid anomaly, Trans. Am. Geophys. Union 60 (1979) 391 (abstract).