Geological Sciences 455: Geochemistry

Problem Set 8 — Due Nov 30, 2007

1. (a) Show that equation 8.52 is identical to equation 8.37 if D (daughter) refers to ^{230}Th, P (parent) refers to ^{238}U and the following assumptions hold:
   $\lambda_{230} >> \lambda_{238}$, $^{238}U \cong {}^{238}U°$, and $e^{-\lambda_{238}t} \cong 1$.
   (b). How valid are these assumptions? (compute them to answer this).

(a) Equation 8.37 is:

$$N_D = \frac{\lambda_P}{\lambda_D - \lambda_P} N_P^0 \left(e^{-\lambda_p t} - e^{-\lambda_D t}\right) + N_D^0 e^{-\lambda_D t}$$

If $\lambda_{230} >> \lambda_{238}$, then $\lambda_{230} - \lambda_{238} \cong \lambda_{230}$ and the equation becomes:

$$N_D = \frac{\lambda_P}{\lambda_D} N_P^0 \left(e^{-\lambda_p t} - e^{-\lambda_D t}\right) + N_D^0 e^{-\lambda_D t}$$

Multiplying both sides by λ_D, we have:

$$\lambda_D N_D = \lambda_P N_P^0 \left(e^{-\lambda_p t} - e^{-\lambda_D t}\right) + \lambda_D N_D^0 e^{-\lambda_D t}$$

The product λN is simply the activity, so the equation becomes:

$$({}^{230}Th) = ({}^{238}U)^0 \left(e^{-\lambda_p t} - e^{-\lambda_D t}\right) + ({}^{230}Th)^0 e^{-\lambda_D t}$$

From our assumptions that: $e^{-\lambda_{238}t} \cong 1$ and $^{238}U \cong {}^{238}U°$, we have:

$$({}^{230}Th) = ({}^{238}U)\left(1 - e^{-\lambda_D t}\right) + ({}^{230}Th)^0 e^{-\lambda_D t}$$

Dividing by the activity of ^{232}Th (and since ^{232}Th is long-lived, $^{232}Th = {}^{232}Th°$), we have:

$$\frac{({}^{230}Th)}{({}^{232}Th)} = \frac{({}^{238}U)}{({}^{232}Th)}\left(1 - e^{-\lambda_D t}\right) + \left(\frac{{}^{230}Th}{{}^{232}Th}\right)^0 e^{-\lambda_D t}$$

which is identical to 8.52. QED.

(b) checking our assumptions

$\lambda_{230} = 9.217 \times 10^{-6} >> \lambda_{238} = 1.551 \times 10^{-10} y^{-1}$

For the rest, let's assume time of the order of 10 half lives of ^{230}Th, i.e., 750,000 years. In this case,

$e^{-\lambda_{238}t} = 0.99988 \cong 1$. ^{238}U will decay according to $^{238}U \cong {}^{238}U° e^{-\lambda_{238}t}$, so $^{238}U \cong 0.99988\,{}^{238}U°$.

2. The following were measured on a komatiite flow in Canada. Use simple linear regression to calculate slopes. Plot the data on isochron diagrams.

	$^{206}Pb/^{204}Pb$	$^{207}Pb/^{204}Pb$	$^{208}Pb/^{204}Pb$
M665	15.718	14.920	35.504
M654	15.970	14.976	35.920
M656	22.563	16.213	41.225
M663	16.329	15.132	35.569
M657	29.995	17.565	48.690
AX14	32.477	17.730	49.996

AX25	15.869	14.963	35.465
M667	14.219	14.717	33.786
M666	16.770	15.110	35.848
M668	16.351	15.047	36.060
M658	20.122	15.700	39.390

a. Calculate the Pb-Pb age and the error on the age. (*Hint: use the Solver in Excel*).
b. Calculate the Th/U ratio of the samples.

The first step is to calculate the 207/204-206/204 and 208/204-206/204 slopes and the error on the slopes. I used the LINEST function in Excel to do this. LINEST returns an array containing a variety of regression parameters. The function INDEX can be used to select the individual values. INDEX(LINEST(y-array,x-array,TRUE, TRUE),1,1) returns the slope and INDEX(LINEST(y-array,x-array,TRUE, TRUE),2,1) returns the error on the slope. Equation 8.34 cannot be solved directly for t, so we have to use an indirect method such as the Solver in Excel to obtain t as a function of the slope. To obtain the + and – errors on the age, we add the error to the slope, then compute t he error for the +age, and subtract the error from the slope to computer the -error (because the equation is non-linear, these errors are not the same, although the difference in this case is small). Once we have t, kappa can be calculated by solving equation 8.35. The spreadsheet is shown below.

		slope	0.173346841	0.899977988
		±	0.003777308	
		+ slope	0.177124149	
lam232	0.049475	-slope	0.169569533	
lam235	0.98485			
lam238	0.155125			Δcalc-obs
t_ Ga	**2.590225608**	calc slope	0.173347193	3.51425E-07
+error Ga	**0.035903719**	calc +slope	0.177124551	4.02378E-07
-error Ga	**0.036831417**	calc -slope	0.169569109	-4.24627E-07
kappa	**3.255159654**			

3. A mafic gneiss from Antarctica has $^{143}Nd/^{144}Nd$ and $^{147}Sm/^{144}Nd$ of 0.511938 and 0.1467, respectively. The present chondritic $^{143}Nd/^{144}Nd$ and $^{147}Sm/^{144}Nd$ are 0.512638 and 0.1967, respectively. The decay constant of ^{147}Sm is 6.54×10^{-12} Ga^{-1}. Calculate the τ_{CHUR}, i.e., crustal residence time relative to a chondritic mantle, for this gneiss.

This problem, in essence, asks us to find the intersection between two lines: the chondritic growth curve and the growth curve for the sample. The sample evolution is described by:

$$(^{143}Nd/^{144}Nd)_{sam} = (^{143}Nd/^{144}Nd)_0 + (^{147}Sm/^{144}Nd)_{sam}(e^{\lambda t} - 1)$$

A similar equation describes chondritic growth. We want to know the value of t when the lines intersect, i.e., when $(^{143}Nd/^{144}Nd)_0$ for both are equal. So solve each equation for the initial value, set them equal to each other, and solve for t (Example 8.3). The result is that t_{CHUR} = 2.13 Ga.

4. What would you predict would be the ratio of the diffusion coefficients of H_2O and D_2O in air?

The ratio of diffusion coefficients should be the inverse ratio of the square root of the reduced mass of the species with air, where the reduced mass is given by:

$$\mu = \frac{1}{1/m_1 + 1/m_2} = \frac{m_1 m_2}{m_1 + m_2}$$

Using 28.8 as the mean molecular weight of air, the reduced masses for H_2O-air and D_2O-air are 11.08 and 11.80 respectively. The calculated ratio of diffusion coefficients is 1.032 (H_2O diffusing faster).

5. A granite contains coexisting feldspar (3% An content) and biotite with $\delta^{18}O_{SMOW}$ of 9.2‰ and 6.5‰ respectively. Using the information in Table 9.2, find the temperature of equilibration.

From Table 9.2 we can write equations for the fractionation between feldspar and quartz and biotite and quartz. These are:

Qz-Feldspar $\Delta = 0 + ([0.97 + 1.04(An)] \times 10^6)/T^2$

where in this case An is 0.03.

Qz-Biotite $\Delta = -0.60 + (3.69 \times 10^6)/T^2$

If quartz is in equilibrium with both feldspar and biotite, biotite and feldspar must be in equilibrium with each other. So we can get an equation describing the fractionation between biotite and feldspar by subtracting the two equations above:

$\Delta_{Biot\text{-}Feld} = 0.60 + ([0.97 + 1.04(0.03) - 3.69] \times 10^6)/T^2 = 0.60 - 2.689 \times 10^6/T^2$

Rearranging, we have:

$$T = \ T = \sqrt{\frac{-2.689\times10^6}{D_{Biot\text{-}Feld} - 0.60}} = 902\ K = 629°C$$

6. Glaciers presently constitute about 2.1% of the water at the surface of the Earth and have a $\delta^{18}O_{SMOW}$ of ≈ –30. The oceans contain essentially all remaining water. If the mass of glaciers were to increase by 50%, how would the isotopic composition of the ocean change (assuming the isotopic composition of ice remains constant)?

Geological Sciences 455: Geochemistry

PROBLEM SET 8 DUE NOV 30, 2007

Isotope fractionation will change the isotopic composition of the ocean, but the isotopic composition of the oceans plus glaciers, i.e., the system, will remain constant. Hence we can write a mass balance equation as follows:

$$\delta_{ocean} M_{ocean} + \delta_{glaciers} M_{glaciers} = \delta_{system} M_{system}$$

or:

$$\delta_{ocean} \frac{M_{ocean}}{M_{system}} + \delta_{glaciers} \frac{M_{glaciers}}{M_{system}} = \delta_{system}$$

Today, or course, the $\delta^{18}O$ of the ocean is 0, $M_{glaciers}/M_{system}$ = 0.021 and if $\delta^{18}O_{glaciers}$ = -30‰, then $\delta^{18}O_{system}$ = -6.3‰. If the glacial mass increases by 50%, then $M_{glaciers}/M_{system}$ = 0.0315, M_{ocean}/M_{system} = 0.9685, and it is a straightforward matter to solve the above equation for $\delta^{18}O_{ocean}$. Substituting values, we find the answer is +.35‰.