Geological Sciences 455: Geochemistry

Problem Set 7 Due Nov 14, 2007

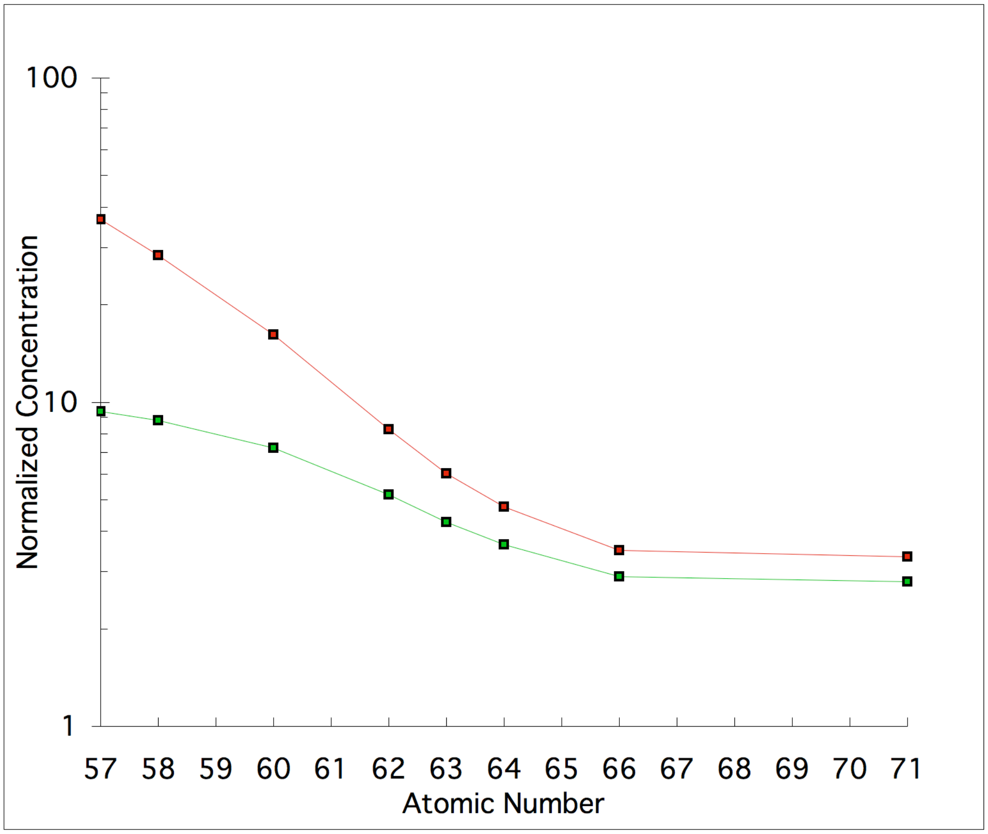
1. Using the Blundy and Wood model, calculate the partition coefficients for the alkali metals in plagioclase at 1250° C. Assume that the site radius in 124 pm and that an ion of this radius would have a partition coefficient of 1. Assume that the value of Young's Modulus in plagioclase is 64 GPa and that ionic radii for the alkali are as follows: Li: 92 pm, Na 118 pm, K 151 pm, Rb 161 pm, and Cs 180 pm.

This problem is a straightforward application of equation 7.20 with D° = 1. We need be cautious of units, however. The easiest thing is to convert all units to SI units (e.g., to pascals and meters). Results are in the Table below:

	r, pm	ΔG	D
Li	92	-2.55E+04	0.133911771
Na	118	-1.05E+03	0.920702834
K	151	-2.51E+04	0.138106157
Rb	161	-4.93E+04	0.020398117
Cs	180	-1.23E+05	6.27791E-05

2 Calculate the enrichment in rare earth elements in an equilibrium partial melt of a mantle consisting of 10% cpx, 5% gar, 25% opx and 60% ol, assuming *modal* (phases enter the melt in the same proportion as they exist in the solid) batch melting for F = 0.02 and F = 0.10. Assume the concentrations in the mantle are chondritic. Use the partition coefficients given in Table 6.5. Use only the following 8 rare earths: La, Ce, Nd, Sm, Eu, Gd, Dy, and Lu. Plot the results on a semi-log plot of chondrite-normalized abundance vs atomic number (i.e. typical REE plot). Draw a smooth curve through the REE, interpolating the other REE. (Hint: *work only with chondrite-normalized abundances, don't worry about absolute concentrations, so the C^0 values will all be 1*).

	cpx	opx	ol	gar	A	Dm	0.02	0.1
La	0.052	0.0056	0.0000088	0.016457		0.00742528	36.66122634	9.373586463
Ce	0.108	0.0058	0.000019	0.06558		0.0155114	28.40814505	8.774988755
Nd	0.227	0.007	0.0001	0.36360		0.04266	16.17944951	7.225746781
Sm	0.462	0.0085	0.000445	1.162		0.103592	8.229087256	5.175104848
Eu	0.458	0.0078	0.001	2.0263		0.14935	6.010951954	4.265938613
Gd	0.55	0.011	0.002	2.764		0.19395	4.760295329	3.642257471
Dy	0.71	0.015	0.006	3.8866		0.27235	3.485498583	2.897584863
Lu	0.623	0.042	0.04	3.7871		0.2858	3.332400261	2.799395331



3. Calculate the *relative enrichments* of La and Sm and the La/Sm ratio in an aggregate melt produced by continuous melting. Assume bulk distribution coefficients for these two elements of 0.01 and 0.05 respectively. Do the calculation for porosities (ϕ) of 0.001 and 0.01.

a.) Plot your results as a function of extent of melting, F, letting F vary from 0.001 to 0.1.

b.) Plot your results on a process identification diagram, i.e., plot La/Sm vs. La, assuming initial La and Sm concentrations of 1.

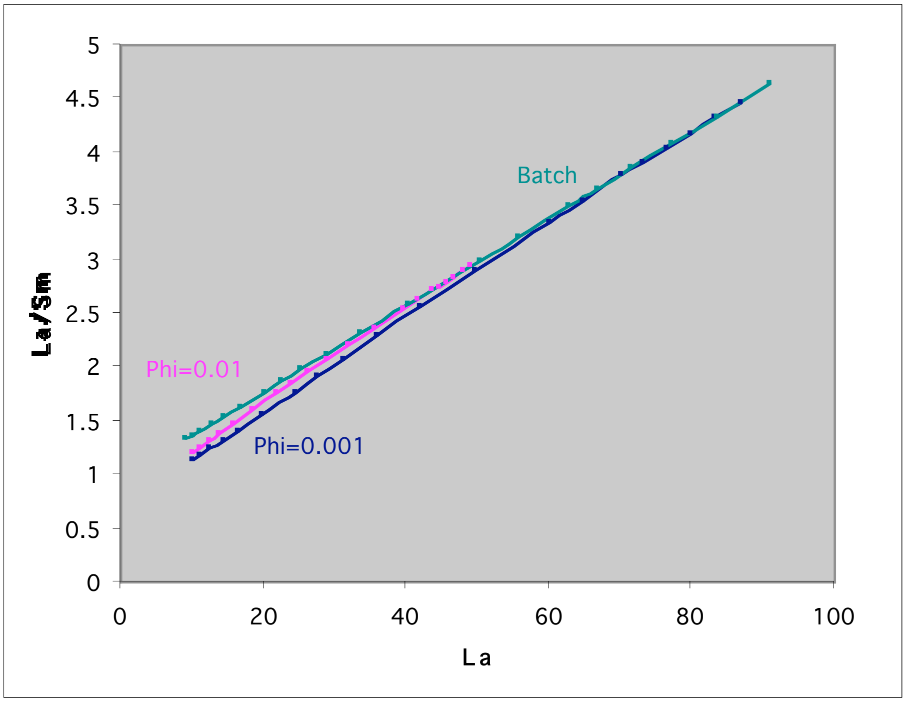
c.) Do the same calculation for batch melting. Compare the two processes, aggregate continuous and batch on a plot of plot La/Sm vs. La.

To answer this question, we just need to apply the relevant equations in Chapter 7. For aggregate continuous melting, we use equation 7.41 (aggregate fractional), but substituting D′ as defined in equ. 7.49, for D. Batch melting is given by equ. 7.39.

Aggregate continuous melting produces enrichments similar batch if ϕ is small, but smaller enrichements if ϕ is large.

The spreadsheet and graphs are shown on the following pages.

						D' phi .001	D' phi .01		
D'=(1-phi)*D+phi				DLa	0.01	0.01099	0.0199		
Cl-bar/Co=(1-(1-f)^(1/D))/F				DSm	0.05	0.05095	0.0595	Cl/Co=1/(D*(1-F)+F)	
		phi=.001			phi=.01			batch	
F	La	Sm	La/Sm	La	Sm	La/Sm	La	Sm	La/Sm
0.001	87.02	19.445	4.4749	49.033	16.675	2.94062	90.9918	19.627	4.636
0.002	83.27	19.266	4.3221	47.854	16.544	2.89259	83.4725	19.268	4.3322
0.003	79.73	19.088	4.1771	46.712	16.414	2.84584	77.101	18.921	4.0748
0.004	76.4	18.913	4.0395	45.605	16.286	2.80032	71.6332	18.587	3.8539
0.005	73.25	18.739	3.9089	44.533	16.159	2.756	66.8896	18.265	3.6622
0.006	70.28	18.568	3.7848	43.495	16.033	2.71286	62.7353	17.953	3.4944
0.008	64.81	18.231	3.5551	41.513	15.785	2.62994	55.8036	17.361	3.2143
0.01	59.93	17.902	3.3475	39.652	15.542	2.55132	50.2513	16.807	2.9899
0.015	49.81	17.112	2.911	35.473	14.954	2.37205	40.2414	15.564	2.5855
0.02	42.05	16.367	2.5689	31.884	14.395	2.21489	33.557	14.493	2.3154
0.025	36	15.664	2.2986	28.792	13.863	2.07697	28.777	13.559	2.1223
0.03	31.25	15	2.0832	26.12	13.355	1.95579	25.1889	12.739	1.9773
0.035	27.45	14.373	1.9102	23.803	12.872	1.84919	22.3964	12.012	1.8645
0.04	24.39	13.78	1.77	21.786	12.411	1.75532	20.1613	11.364	1.7742
0.05	19.81	12.692	1.561	18.481	11.554	1.59948	16.8067	10.256	1.6387
0.06	16.61	11.719	1.4171	15.923	10.775	1.47771	14.4092	9.3458	1.5418
0.07	14.27	10.848	1.3152	13.913	10.067	1.38209	12.6103	8.5837	1.4691
0.08	12.49	10.067	1.2411	12.311	9.4218	1.30662	11.2108	7.9365	1.4126
0.09	11.11	9.3659	1.1861	11.014	8.834	1.24676	10.0908	7.3801	1.3673
0.1	9.999	8.7355	1.1447	9.9498	8.298	1.19907	9.17431	6.8966	1.3303


La/Sm
La
Batch
Phi=0.01
Phi=0.001
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
20
40
60
80
100

4. What are the binding energies per nucleon of ^{147}Sm (mass = 146.914907 u) and ^{143}Nd (mass = 142.909823)?

masses

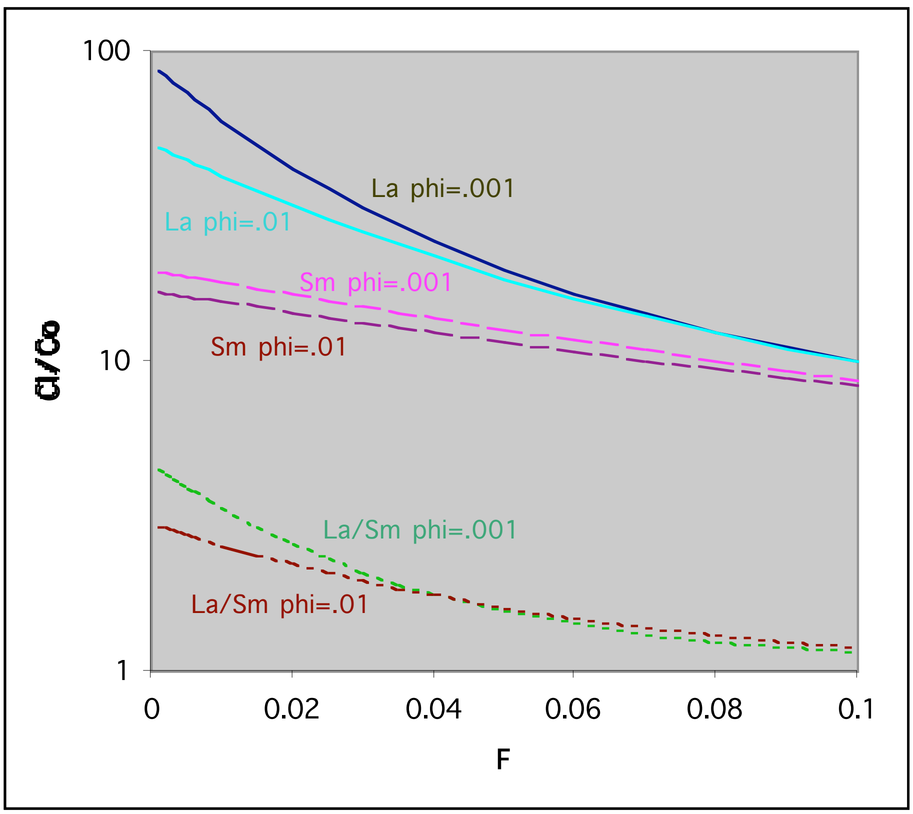
proton	1.007593
neutron	1.008982
electron	0.000548756

	W (u)	M (u)	δ (u)	δ (kg)	δ/A (kg/nucleon)	E_b (J/nucleon)	E_b (MeV)
^{143}Nd	144.2340114	142.91	1.32401136	2.198E-27	1.54E-29	1.38E-12	13.00775961
^{147}Sm	148.2682589	146.91	1.358258872	2.255E-27	1.53E-29	1.38E-12	12.9811164

5. What is the decay constant (λ) of ^{152}Gd if its half-life is 1.1×10^{14} yr?

The decay constant is simply $\lambda = \ln(2)/t_{1/2} = 6.3 \times 10^{-15}$.

6. The following were measured on a komatiite flow in Canada. Use simple linear regression to calculate slope. Plot the data on isochron diagrams.
a. Calculate the Sm-Nd age.



b. Calculate the *initial* ε_{Nd}.

	$^{147}Sm/^{144}Nd$	$^{143}Nd/^{144}Nd$
M654	0.2427	0.513586
M656	0.2402	0.513548
M663	0.2567	0.513853
M657	0.2381	0.513511
AX14	0.2250	0.513280
AX25	0.2189	0.513174
M666	0.2563	0.513842
M668	0.2380	0.513522

The slope and error on the slope can be calculated in excel using the SLOPE and INDEX(LINEST(Y,X,TRUE, TRUE),2,1) functions. From these, we calculate the age according to equation 8.21. The initial 143/144 ratio is calculated using the INTERCEPT function. The error on the intercept can be obtained from INDEX(LINEST(Y,X,TRUE, TRUE),2,2).

To calculate ε_{Nd} we use:

$$\varepsilon_{Nd-i} = \frac{(^{143}Nd/^{144}Nd)_i - (^{143}Nd/^{144}Nd)_{chon(t)}}{(^{143}Nd/^{144}Nd)_{chon(t)}} \times 10^4$$

where the chondritic value is the chondritic value at the time of the initial value, in this case, 2.714 Ga. So we first need to calculate that. We can do this from the present day chondritic 143/144 and $^{147}Sm/^{144}Nd$ and by solving "isochron" relationship:

$$\left(\frac{^{143}Nd}{^{144}Nd}\right)_{chon-today} = \left(\frac{^{143}Nd}{^{144}Nd}\right)_{chon-2.7Ga} + \left(\frac{^{147}Sm}{^{144}Nd}\right)_{chon-today} (e^{\lambda 2.714Ga} - 1)$$

for the chondritic 143/144 at 2.714Ga.

M654	0.2427	0.513586	slope	0.017910	
M656	0.2402	0.513548	error	0.000194	
M663	0.2567	0.513853	age	**2.714**	**Ga**
M657	0.2381	0.513511	error ±	0.0291	Ga
AX14	0.225	0.51328	initial	**0.50925**	
AX25	0.2189	0.513174	error ±	4.64564E-05	
M666	0.2563	0.513842	chon @2.714	0.50912	
M668	0.238	0.513522	eNd	**2.655**	
			error ±	0.912	